Voices of Mathematicians and Mathematics Teacher Educators Co-Teaching a Mathematics Course for Prospective Secondary Teachers

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Both mathematicians and mathematics teacher educators (MTEs) have responsibility for preparing preservice mathematics teachers (PSTs). In many institutions, mathematics content courses are taught by mathematicians, and mathematics pedagogy courses by MTEs. In separate departments or colleges, these two groups often live in different worlds with different cultural norms. The Conference Board of the Mathematical Sciences (CBMS, 2001) notes: "There is considerable distrust between mathematics faculty and mathematics education faculty both within institutions and through public exchange. Conscious efforts … are needed to foster cooperation, along with mutual understanding and respect" (p. 9).

Public discourse about mathematics teacher preparation is often based on content knowledge. Yet as Ball (2003) acknowledges, "increasing the quantity of teachers' mathematics coursework will only improve the quality of mathematics teaching if teachers learn mathematics in ways that make a difference for the skill with which they are able to do their work." Engaging teachers in learning mathematics in ways that resonate with future expectations for teaching suggests a need for collaboration between mathematicians and MTEs as recommended by the CBMS:

> Most good school mathematics instruction involves a combination of mathematical knowledge and pedagogy …. Mathematics educators can provide valuable insights and information about what takes place in school classrooms, including common mathematical misunderstandings of practicing teachers. … [M]athematics faculty can help mathematics education faculty by keeping them informed of mathematical developments which have an impact on school mathematics. (p. 9)

Collaboration can take many forms, from sharing of ideas and philosophies in occasional discussions (Ball et al., 2005) to developing curriculum materials for professional development with teachers (Kersaint & Berger, under review) to co-teaching of courses (Grassl & Mingus, 2007). In this paper, we share our experiences in creating a learning community from a collaboration among a mathematician (Catherine), two MTEs (Denisse and Gladis), and a

mathematics education doctoral student (Sarah) who collaborated on the development and delivery of a geometry course required of all secondary PSTs.

We start by describing the context for the collaboration and some practical logistics, including our preparation for class and experiences we designed for PSTs. Then, we each offer our perspective on the learning community that emerged among the four of us. We also share insights from PSTs collected during a focus group discussion about how they viewed the roles of the two instructors and how their experiences in our class compared to their experiences in other mathematics courses. We conclude by looking across the perspectives of the four collaborators for common views and issues related to such endeavors.

Context for the Collaboration

The University of South Florida is a research university with over 47,000 students. Although the mathematicians and MTEs coordinate class schedules to avoid conflicts for PSTs, more in-depth collaborations have only begun within the last five years. Gladis has had several grants in which mathematicians, including Catherine, have engaged to develop and deliver professional development for teachers, K-12. This work laid the seeds for the collaboration discussed here.

Our university is one of four universities participating in a NSF-funded project based at the University of Arizona[1] studying the nature of collaboration when a mathematician and a MTE co-teach a mathematics content course and a mathematics pedagogy course. Although Gladis secured our participation, she was not able to serve as co-instructor, so Denisse fulfilled this role. Hence, an opportunity arose for a unique collaboration of four individuals – Catherine and Denisse as co-instructors for two courses – and Gladis and Sarah who participated in the planning and observed most classes. The collaboration described in this paper occurred during

[1] NSF DR K-12 #0821996 Knowledge for Teaching Secondary School (KnoTSS), with PI Rebecca McGraw.

the Fall 2009 semester.

Neither Catherine nor Denisse had ever previously taught this college-level geometry course nor had they previously collaborated in any way. The course met 75 minutes twice each week. Individual desks were arranged in rows, and a computer smart system and document camera were permanently in the room.

Overall Goals of the Course and Collaboration

When we first met to design a syllabus, we established some objectives that would permeate the course. Specifically, PSTs should

- learn mathematics using inquiry-based approaches as recommended by the mathematics education community (e.g., Martin, 2007; NCTM, 2000);
- reason about and make sense of mathematics for themselves, often within a structure of collaborative groups;
- write mathematical proofs, and use the language of mathematics appropriately.

Throughout the semester, the four of us met at least once per week for two hours to prepare for upcoming classes. ,. We typically began by sharing insights from the previous week's classes, specifically sharing observations about instruction, concepts that seemed to cause difficulty, or particularly noteworthy comments from PSTs. We then discussed the content for the week and created PowerPoint outlines in which we indicated activities that were to occur, theorems to be stated or proved, and teaching notes for ourselves regarding important points. The planning meetings were an opportunity to brainstorm inquiry-based activities for engaging PSTs with the content and to raise concerns or questions about the content itself. In every class, we made an effort to incorporate some kind of group work.

To ensure that both Denisse and Catherine truly co-taught the course and shared significant

instructional roles in each class period, we designated in our notes who would lead each segment of the lecture or class activity. Thus, we created an expectation for ourselves that Denisse and Catherine had a significant contribution to make during every class

A Look into the Class: Typical Activities

First Day. Setting the tone for the semester on the first day of class was crucial. We wanted to establish both for ourselves and for the PSTs that mathematics as well as education issues would be present and that a social environment would be created in which PSTs would discuss the content and work with each other. So, we began this class by having the PSTs put themselves in order according to their birthdays (day and month) without speaking. Such an activity strongly hinted that this class might be different from what they normally experienced in mathematics! We had the PSTs discuss why we had engaged in such an exercise, and they raised issues of communication, group work, and their own expectations for a mathematics class. The activity provided a shared experience for all to serve as a foundation for what would occur throughout the semester.

A typical class day. For each class, PSTs were expected to read some portion of their text prior to class, often focusing on key definitions related to geometry. Weekly, we gave a brief quiz to support this reading and also collected homework. It was typical to begin class with feedback from a previous class, such as topics from a previous lecture that seemed unclear to PSTs or common mistakes in the homework or on the quizzes. We then introduced the day's work or activity, such as examining or discovering definitions or proofs of theorems.

Sample strategies. Throughout the semester, we experimented with various activities to engage PSTs during group collaboration. Because geometry is highly visual, we often used patty paper to illustrate concepts or to support developing proofs. For example, on one occasion, PSTs

had read prior to class about important lines in a triangle (i.e., median, altitude, angle bisector). Then, given a triangle, they used patty paper to construct a median of a triangle. For instance, they folded the patty paper so that two endpoints of one of the sides of the triangle matched and made a crease; then the point at the crease represented the midpoint of that side. As PSTs drew the segment from the opposite vertex to this midpoint, their actions reinforced the distinction between "midpoint" and "median" in a concrete manner without that distinction needing to come from either instructor. On another occasion, we used patty paper to model the Side-Angle-Side congruence test; by physically moving triangles and lining up sides and angles, PSTs had a hands-on experience of the meaning of this important congruence test.

Another particularly successful strategy that emphasized the importance of communication was to have PSTs critique, anonymously, written work of their peers. Specifically, we selected sample responses to a quiz or a homework problem and prepared these on a handout. PSTs worked in groups to evaluate the responses according to the following guidelines: (1) Were the responses correct, clear, complete? (2) If the response was incorrect, could it be suitably modified? After PSTs discussed these sample responses within their groups, we had a whole-class discussion of their critiques. This exercise seemed to help PSTs understand our expectations for clear written work without those criticisms coming directly from us. The PSTs were typically thorough in their critiques, so the instructors rarely had to make additional comments but simply had to guide the discussion for clarity. At the end of the exercise, PSTs had coherent, concise, and correct answers to the given questions and ownership over the material in a way that might not have occurred had they simply received a corrected paper.

Grading and evaluation. Denisse and Catherine both fully participated in grading the PSTs' work. Although grading of assignments alternated between instructors, they discussed the graded

papers and made any adjustments before returning assignments. This shared responsibility for grading gave each instructor ownership of the course and was key to the collaboration being significant. PSTs generally could not tell who graded their assignments.

Sharing our perspectives on the collaboration

In this section, we share our individual perspectives on the collaboration. We start with the two co-instructors (Catherine, the instructor of record, and Denisse), followed by the two mathematics educators who observed the class (Gladis and Sarah).

Catherine speaks as a mathematician co-teaching the course. I had never team taught a course before; sharing the privacy and intellectual domain of the classroom was initially difficult. Having observers (Gladis and Sarah) made me particularly self-conscious, and I had to adjust to receiving constructive criticism. Because I had not taught geometry before, I had few pre-conceived notions about the course itself and was willing to keep an open mind about topics to cover or strategies to try. After a few weeks, I began to enjoy the natural cycle of feedback and discussion about what went well and what didn't. One day when Denisse couldn't make it to class and the others were late, I was disappointed to be on my own; I had become accustomed to sharing the classroom and bouncing ideas off her during class.

One of the things I most enjoyed about the collaboration was the preparation for class. We each reviewed the mathematics in the sections we planned to cover before our meeting, shared ideas about what concepts were important, and brainstormed about how they should be taught and the activities we might incorporate. We spent time working challenging geometry problems. Because we were using a traditional mathematics text, we struggled to identify ways of teaching the material that would support an inquiry approach. I found the mathematical and pedagogical challenges of the preparation intellectually stimulating.

Before I began this collaboration, I considered myself a "good" teacher and someone who connects to students and recognizes what they do not understand. I had previously experimented with various strategies (e.g., group work, technology), but I feel that there was something fundamentally different about the initial setup of this geometry class. Although the strategies we employed were not that different from those I had implemented on occasion in other classes, they were incorporated as a fundamental piece of the geometry course design.

This collaboration has given me numerous concrete ideas about how to deliver instruction with a student-centered approach, such as having students use patty paper in geometric constructions or spaghetti to discover the triangle inequality. One specific technique I found useful was to create a handout of students' quiz responses for them to critique. My role shifted to discussion leader, and criticisms came from students, who were able to judge, with a little guidance, how what was written was not coherent or correct, or needed some adjustment. Through such activities, students became more critical during the semester, better able to evaluate peers' responses, and more adept at writing their own responses in future assignments. In addition, Denisse and I gained access to what the students really understood. It became clear that students were having great difficulty in identifying the hypothesis and conclusion of a given mathematical statement, and we needed to spend time helping them understand references to any pronouns in the statement of a theorem before we had any hope of teaching students to prove it.

I also became familiar with what students themselves will face in the classroom in terms of content, and more importantly, I gained a sense from my collaborators about what is emphasized in the high school curriculum, such as the use of congruence tests for triangles. This is difficult for a mathematician to know if working alone.

Throughout the semester, I became more aware of how my own imprecise use of notation or

language might affect student learning. For example, I was not careful to write relationships for similar triangles so the vertices corresponded to the angles that are congruent. Although I would never mark a response wrong if the correspondences were not written in the proper order, I had not spent much time thinking about how such notation might confuse students.

Finally, the time commitment was tremendous. I wouldn't have been able to participate in this collaboration before I applied for tenure, because it took time away from more traditional mathematics research activity. Although my chair has been supportive, in terms of "research deliverables" that carry weight in a mathematics department, this project might not be considered valuable. On a personal level, however, the project has been stimulating and has changed my view of myself as a teacher and what my classroom could be. I believe this collaboration might be a starting point for interesting discussions in my own department.

Denisse speaks as a mathematics teacher educator co-teaching the course. Geometry has never been my favorite area, even though I taught high school geometry one year. So, being responsible for teaching while being observed was a bit unsettling at the beginning. I felt like judgments might be made about my mathematical knowledge. Even though I had previously team-taught a geometry course for inservice teachers with Gladis, there was still anxiety at the beginning, particularly when the observers saw something that they thought could be improved.

I was concerned about entering into this collaboration because I felt like the odd person out. Gladis and Catherine already had a working relationship; Catherine and Sarah also had a relationship because Catherine had been Sarah's master's thesis advisor. I mention this initial unease because developing trust and mutual respect are essential to an effective collaboration and this develops over time. It builds gradually as each team member shares during planning and potential instructional ideas are discussed, validated, modified, and respected.

I completed my mathematics coursework well before the implementation of the standards movement from NCTM. Even though I try to teach mathematics pedagogy courses with discussions, cooperative group work, and problem solving, I have not had an opportunity to implement those practices into a mathematics content class. I was faced with the practical reality that many of the recommended pedagogical approaches are challenging to implement, especially when you feel that so much content needs to be addressed. Deciding that it is okay to give students more time to discuss a problem in their groups, even though it means that the planned lesson is not finished, is difficult. Deciding that completing fewer content chapters is okay is hard, even though I believe that students have a better mastery of the content that we covered. The goal to have PSTs become careful about their language when writing mathematics meant we needed time to discuss and critique samples of their writing if they were going to improve. The time spent on this activity meant time had to be taken from elsewhere. It is easy to talk about "less is more" from an abstract perspective; it is harder to put that perspective into practice when you are responsible for teachers' mathematics content knowledge.

I am pleased that our collaboration resulted in a classroom in which students regularly engaged in mathematical conversation and in which they experienced learning content using practices I typically discuss in methods classes. Catherine brought good mathematical insight into our discussions and helped ensure that the cognitive demand of the tasks remained high. Even when the PSTs were struggling with content, Catherine was not willing to lessen her expectations but wanted to work together to implement strategies (e.g., the critique of quiz responses) that could help the PSTs overcome those struggles on their own. While wanting to ensure that the essential content was addressed, she was more concerned with PSTs' learning than with rushing through content for the sake of content coverage. Because of past discussions

with students, this was not what I had expected from the mathematicians at my university.

I feel I was an equal partner in teaching the geometry course, but the time commitment was huge. I was not technically listed as an instructor of record. Because we were participating in the grant, I had a course release during the year. But without such a release, could I afford to spend the time that such collaboration requires? In mathematics education, we teach the same courses on a regular basis so time commitments for intense reflection and course modification pay off. In contrast, in our mathematics department, courses circulate among various faculty members. So, I could invest the time to suggest improvements in one of the content courses taken by our PSTs and it might be for naught when someone else teaches the course in the next semester. Regardless of the outcome, I learned that it is important for MTEs to take an active role in enhancing mathematic content courses, particularly those taken by prospective teachers.

Gladis speaks as a mathematics teacher educator observing the course. Unlike Catherine and Denisse, I spent several years teaching high school geometry using many of the strategies recommended for instruction by the mathematics education community, including the use of technology. So, I was looking forward to my role as a co-instructor and to incorporating those strategies in teaching a university content course. Although I couldn't be a co-instructor, I thought it would still be possible to incorporate many of those approaches through my participation in the planning. However I found this to be a challenge.

Because I was familiar with all the players, I assumed trust would exist from the onset. Denisse and I had collaborated in various capacities, including co-teaching a course, co-authoring manuscripts, and collaborating on programmatic issues. Catherine had been involved in a prior grant effort to develop and deliver content-specific professional development which resulted in my observing her teaching. Sarah was a doctoral student in one of my courses.

However, I failed to anticipate the need for others to cultivate relationships. This was clearly an oversight given that Catherine and Denisse met only to engage in this effort. In hindsight, it makes sense that each approached this effort with some trepidation.

During initial planning meetings, I shared what I observed to facilitate planning for future sessions. As an observer, I was able to focus on instructional issues, instructor-student and student-student interactions, and the nature of the learning environment. During meetings, I shared what I observed as a means to facilitate planning for subsequent class sessions. I was surprised that both Denisse and Catherine found this intimidating and initially viewed observational comments as judgmental. Denisse and Catherine had no prior working relationship and were grappling with the nature and content of the course (e.g., how it was addressed in the selected text, what content to emphasize).Having an observer comment on every aspect of the classroom was jarring when they were trying to figure out how best to work with each other. Consequently, I had to adjust my role to allow the instructors to make sense of the course on their own terms. So, I became selective in observations and strategies I shared. In earlier stages, I suggested instructional approaches that were familiar to me. However, on occasions their use in the classroom revealed a lack of shared meaning among the group that was not apparent during the planning meeting. Although I knew what I would do, say, ask, and emphasize if I were teaching, that information did not transfer as I explained it to the others. At times, Sarah and I discussed whether what was observed represented what had been shared during planning. We both acknowledged that perceived differences were differences in interpretations rather than incorrect uses of any strategy. Viewing these differences highlighted the need for shared meaning among the collaborators during planning. This required that I step back and ask questions to determine what individuals were making of the discussions, such as “What is the

goal?" and "How do you see this playing out in the class?" Responses to these questions revealed differences in interpretations, providing opportunities to clarify understandings.

I am pleased with this collaboration. The classroom environment, students' feedback, and collaborator feedback have been positive. This collaboration has reinforced previous work in engaging mathematicians meaningfully in the work of teacher education. There is greater appreciation of the role both groups play. These initial efforts will broaden the discussion among MTEs and mathematicians and build the foundation for other efforts to improve the mathematical education of teachers.

Sarah speaks as a mathematics education doctoral student observing the course. I have studied in both the mathematics and the education departments at this university. In universities like ours, PSTs often experience a disjointed education. Mathematicians are responsible for PSTs' learning of mathematical content, and MTEs are responsible for PSTs' learning of mathematical pedagogy. However, it is not often made clear to PSTs how the content of their university mathematics courses, seemingly disparate from the high school mathematics curriculum, relates to their future classrooms. Within this course, Denisse was able to point out issues related to the content in high school geometry classes that would be important for PSTs to know. For example, during a discussion about quadrilaterals, Denisse noted that the definition of trapezoid may differ depending on the text that teachers use, an important consideration given accountability pressures of high-stakes testing in which definitions on tests may be inconsistent with definitions in curricular materials.

PSTs were not only able to make connections between the content of high school and university level mathematics, but were also able to experience inquiry-based instruction within an actual mathematics class, a type of instruction many PSTs have rarely seen put into practice

within mathematics courses. As a student new to the field of education myself, and accustomed to the traditional teacher-centered, lecture-style instruction within the mathematics department, I often feel overwhelmed by the recommendations for inquiry-based teaching and learning. What exactly does it mean to teach in an inquiry-based fashion? Had I ever seen a teacher who taught that way? What would it look like? I can't help but believe PSTs ask themselves the same questions. Within this course, PSTs were able to experience first-hand an inquiry-based classroom. For example, after working on a problem collaboratively in small groups, Catherine and Denisse would pull the PSTs back together for a whole-class discussion in which the PSTs' contributions, as opposed to the instructors', would determine the flow and direction of the discussion. PSTs listened closely to each other, focused on the accuracy and precision of mathematical language used within the classroom, and questioned each other when something was unclear or seemingly incorrect.

I believe this type of classroom environment was successfully cultivated as a direct result of our collaboration. Our differing levels of experience as teachers, mathematicians, and teacher educators contributed to the variety of perspectives through which we viewed the course. As MTEs, Gladis and Denisse identified activities that would help PSTs discover mathematical relationships on their own and develop precision of mathematical language. As a mathematician, Catherine recognized connections among foundational aspects of the subject that helped lead the PSTs derive formulas instead of simply memorizing them (e.g., the law of cosines). As someone new to education, I was eager to learn about pedagogical tools and activities teachers could use in their own classrooms, and therefore researched and proposed several such activities (e.g., breaking up spaghetti into three pieces to formulate a conjecture about the triangle inequality). While Catherine and Denisse were teaching, Gladis and I made observations and discussed the

unfolding events. We viewed the instruction from the perspective of the instructors and also focused our attention on the reactions of the PSTs to the instruction. During our weekly meetings, the four of us discussed class sessions from our different perspectives and used these reflections to design the next class accordingly.

I gleaned several personal lessons from participating in this collaboration. Through interaction with and observation of secondary PSTs, I made valuable connections between research and practice. Because my current research interests focus on the teaching and learning of mathematical proof, being in the classroom with PSTs and reflecting on their learning of proof in geometry helped bring readings from research to life.

Prior to this collaboration, I would have resisted having someone observe my classroom. But now, I welcome the opportunity to have someone with which to reflect on my teaching practices because I see the value that such discussion and reflection can bring to one's instruction.

Perspectives and insights from the preservice secondary teachers

Wu (2006) suggests it is important for mathematicians to become involved in mathematics education, as a way to ensure mathematical precision and because, but also because mathematicians have a wealth of pedagogical knowledge from their own years of teaching in university-level classrooms. As MTEs, we learned that Catherine did have valuable insights about the nature of the mathematics content, the course, and student learning. One of our students echoed this notion:

> *I think Dr. Beneteau's extensive teaching in the math department gives some pretty good insight into how her students learn. It might be different material … calculus and analysis, but she still knows how students learn, what questions they have, how to help them during office hours, during discussions, how to teach them. She teaches well.*

One student voiced concern over the style of instruction in the course, illustrating a preference towards the familiar lecture-style model of instruction over the inquiry model we

used. Others (e.g., Lubienski, 2000) have documented middle school students' apprehension/resistance towards inquiry-style learning environments, which is typically linked to years of enculturation within traditional lecture-style mathematics courses.

> *That's [style of instruction] one of the few things that I did not like about the geometry course. I like math lectures. I don't like group work, I don't work well with others, I like to show up, do my stuff, learn my stuff, be tested on my stuff, and collect my A.*

Other students, however, appreciated the opportunity to learn geometry through an inquiry-based methods and group work.

> *I liked geometry when we did group work. I'm sorry, I'm the kind of person that needs to be engaged …. Like math lectures, I fall asleep. I don't pass the class. I don't do the homework because I don't care about the class. The geometry homework was a lot more engaging because I enjoyed that class a lot and I feel like it was because of the teaching style. I liked it because it was different, because it was group work and stuff.*

Personal insights gained from the inquiry perspective were also evident in one particularly poignant instance. During a class focusing on the distance formula in the plane, PSTs were guided to determine the distance along horizontal and then vertical segments. They then considered what they knew about the Pythagorean theorem to determine the distance along an oblique segment. After the typical distance formula was determined by the PSTs on their own in their group, one PST doing quite well in the class commented, "I feel so dumb." In discussing her comment with the class, she indicated that she had never made the connection between the distance formula and the Pythagorean theorem; she had simply memorized the distance formula and often got it wrong. Her comments and the subsequent discussion provided a unique opportunity to discuss the value of understanding over rote memorization; when one knows how a formula is generated, he or she can quickly reconstruct it when needed.

To many of the PSTs, the lines between co-teaching and inquiry teaching were blurred because the incorporation of a mathematics educator within the mathematics class provided opportunities for them to see and learn the subject in new ways. One PST compared Denisse's

presence in the geometry course to previous experiences she had with her in education courses:

> *She [Dr. Thompson] did some of the things she does also in the methods class, that in a straight math class I've never been exposed to, group work in a college math class … the way that she analyzed our proofs on the board. Having the students write them and then going through them line by line to see what's right, what's wrong, how to improve the proof. I did learn a lot about geometric proof writing in that course and I think Dr. Thompson had a lot to do with that.*

Conclusion and Discussion

To improve the mathematics preparation of future secondary teachers, mathematicians and mathematics teacher educators must work together. All four of us believe the collaboration and the learning community that we developed among ourselves has enhanced our perspective of the role each plays in preparing future teachers. As we reflect on this co-teaching collaboration, common perspectives permeate our individual narratives.

We needed time to develop respect and trust and to build a way of working together. Catherine needed time to recognize that Denisse was both mathematically competent and interested in engaging in the mathematics. Denisse needed time to realize that Catherine was willing to try different pedagogical strategies, even if she was not sure they would work.

The collaboration supported the development of an inquiry classroom (e.g., a constructivist approach) within the learning of a content course. Catherine learned what constructivism was, Denisse was able to implement methods within a content course that she discusses in pedagogy courses, and Sarah witnessed how research about constructivism can be put into practice. Catherine had a chance to realize the benefits of a constructivist classroom to engage students with mathematics; Denisse and Gladis experienced some of the challenges in implementing recommended instructional approaches.

In order for our collaboration to work smoothly and seamlessly, we had to develop shared meanings and a common language. For instance, little acronyms like NCTM or the *Standards*,

used regularly by mathematics educators, are not necessarily understood by mathematicians.

Because neither Catherine nor Denisse had previously taught this geometry course, neither came to the co-teaching with preconceived notions of what the course should look like. They were creating a course together. As indicated in the narratives, Gladis had a long history of teaching geometry and initially had a vision of what should take place instructionally at certain points; as noted, this created some initial discomfort which had to be worked through.

We each had to give up some of our personal space within the classroom as part of the collaboration. Typically, instructors are the ones in control. Because of the collaboration, we each had to share the pedestal and the relationship that instructors build with their students.

The time commitment was immense, much more than either Denisse or Catherine expected. They both engaged in all stages of the collaboration, sharing teaching, planning, and grading. Through grading, each instructor developed relationships with the PSTs, learned about their strengths and misconceptions, and could determine further instructional practices to use. The time commitment resulted in more reflection about the course than often occurs when one teaches alone, and this reflection had to occur during time when all parties were available rather than 2 a.m. in the morning in one's pajamas. The course release supported by the grant made it possible to commit the needed time. Although we believe in the collaboration, trying to make it happen without such support is difficult. Such efforts are not incentivized by our university's reward structures and take time from other areas (i.e., research) that are rewarded. Though we are expected to be effective teachers, the amount of time spent on course preparation in such co-teaching contexts might be considered excessive, regardless of the outcome. When time is limited, faculty have to make choices among good projects. Choices might be different at different stages of one's career; it is often easier for an established faculty member to spend time

on such endeavors than for faculty seeking promotion who need to publish.

We all believe we grew professionally. Denisse and Gladis gained a better appreciation for the concerns that Catherine and other mathematicians have about PSTs' mathematical knowledge. The openness we developed in talking about the mathematical background of our PSTs and their needs provides a foundation on which to build collaborations to improve other courses taken primarily by PSTs. As documented, the PSTs generally had positive perceptions of the course developed through this collaborative endeavor.

Although this paper discusses our learning from a single semester of collaboration, we continued the collaboration during Spring 2010 in a mathematics pedagogy course. Issues about building trust and working through initial tension did not arise the second semester, and the collaboration was smoother. Thus, continued collaboration has the potential to enable more instructional innovations to be incorporated into teacher preparation programs in both content and pedagogy. The challenge is to develop support to institutionalize such endeavors.

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